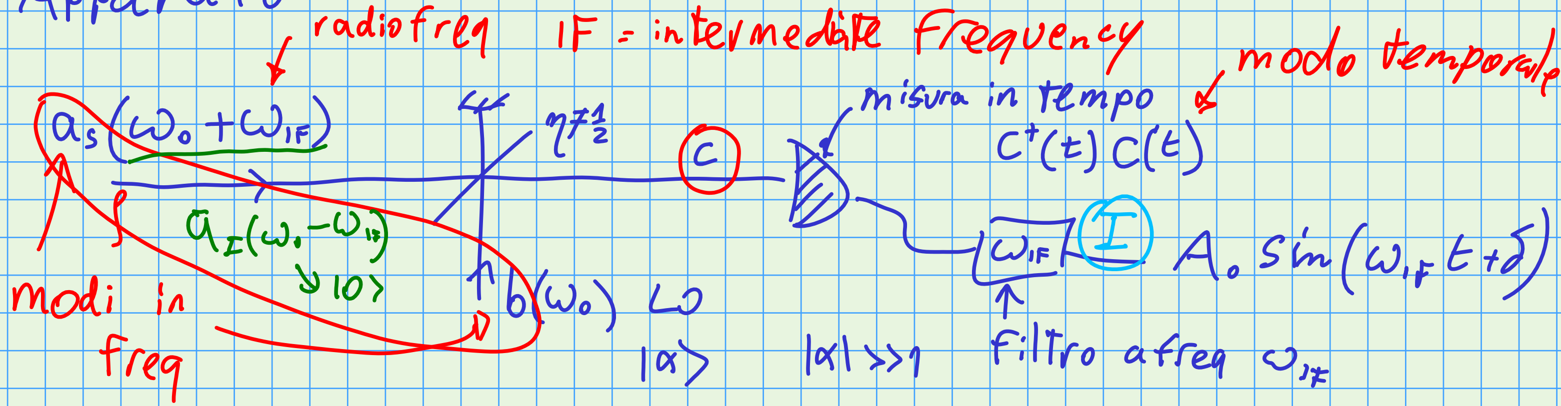
OTTICA Q 17/12/20

ETERODINA $\longrightarrow$ metodo per fare misura congiunta di 2 quadrature coniugate: è una misura congiunta di x e p

Apparato



$\downarrow$

$$a(\omega) = \int dt\, e^{-i\omega t} a(t) \qquad [a(t), a^\dagger(t)] = 1$$

$$a(t) = \int \frac{d\omega}{2\pi} e^{i\omega t} a(\omega)$$

POVM eterodina $\boxed{\Pi_z = |z\rangle\langle z|}$ $\quad z \in \mathbb{C}$ $\qquad x \propto \mathrm{Re}\, z$, $p \propto \mathrm{Im}\, z$

$$P(z) = \mathrm{Tr}\{\rho\, \Pi_z\} = P(\mathrm{Re}\, z, \mathrm{Im}\, z)$$

uscita dopo il filtro

$$I(\omega_{IF}) = \int dt\, e^{i\omega_{IF} t} I(t) =$$

guardo solo la comp ω_{IF} del segnale $I(t)$ $\qquad$ misuro in tempo $I(t) = c^\dagger(t)\,c(t)$

$c(t) = \int \frac{d\omega'}{2\pi} c(\omega') e^{+i\omega' t}$

$$= \int dt\, e^{i\omega_{IF} t} c^\dagger(t)\, c(t) = \int dt\, e^{i\omega_{IF} t} \int \frac{d\omega\, d\omega'}{4\pi^2} c^\dagger(\omega)\, c(\omega')$$

$$\times\, e^{\pm i(\omega-\omega')t}$$

$$= \left(\int \frac{d\omega'}{2\pi}\, c^\dagger(\omega'+\omega_{IF})\, c(\omega')\right) = I(\omega_{IF})$$

$$\int dt \longrightarrow 2\pi\,\delta(\omega-\omega'-\omega_{IF})$$

$$\int dt\, e^{iat} = 2\pi\,\delta(a)$$

$$c(\omega) = \sqrt{\eta}\, a(\omega) + \sqrt{1-\eta}\, b(\omega)$$

limiti $\left.\begin{array}{l} |\alpha| \to \infty \\ \eta \to 1 \end{array}\right\}$ in modo t.c. $\gamma \overset{def}{=} |\alpha|\sqrt{\eta(1-\eta)}$ è costante

$$I(\omega_{IF}) = \int\frac{d\omega'}{2\pi}\overbrace{\left[\sqrt{\eta}\, a^\dagger(\omega+\omega_{IF}) + \sqrt{1-\eta}\, b^\dagger(\omega+\omega_{IF})\right]}^{c^\dagger(\omega+\omega_{IF})} \times \underbrace{\left(\sqrt{\eta}\, a(\omega) + \sqrt{1-\eta}\, b(\omega)\right)}_{c(\omega)} =$$

$|\alpha\rangle$ ha freq ω_0 ed è intenso
$\Downarrow$
$\int d\omega'$ sopravvivono solo i termini $b(\omega_0)$

$$\sim \frac{\sqrt{\eta(1-\eta)}}{2\pi}\Big(\underbrace{a^\dagger(\omega_0+\omega_{IF})}_{\overset{def}{=}\, a_s^\dagger} b(\omega_0) + b^\dagger(\omega_0)\underbrace{a(\omega_0-\omega_{IF})}_{\overset{def}{=}\, a_I}\Big) + (1-\eta)\Big(b^\dagger(\omega_0)\, b(\omega_0-\omega_{IF}) + \dots\Big) = I(\omega_{IF})$$

$$Z \overset{def}{=} {}_b\langle\alpha| \lim_{\substack{\eta\to 1\\ |\alpha|\to\infty \\ \gamma = cost}} \underbrace{\gamma^{-1}}_{= |\alpha|\sqrt{\eta(1-\eta)}} I(\omega_{IF})\, |\alpha\rangle_b =$$

$$= {}_b\langle\alpha|\, a_s^\dagger\, b(\omega_0) + b^\dagger(\omega_0)\, a_I\, |\alpha\rangle_b + \cancel{\underbrace{\lim_{\eta\to1}(1-\eta)}_{=0}\langle \quad | \quad \rangle}$$

$|\alpha|$

$$= \frac{a_s^\dagger \alpha + a_I \alpha^*}{|\alpha|} \underset{\alpha = |\alpha| e^{i\varphi}}{=} a_s^\dagger e^{i\varphi} + a_I e^{-i\varphi} \underset{\text{scelgo } \varphi = 0}{=}$$

modo a_I è nel vuoto

$$\boxed{\hat{z}_1 = a_s^\dagger + a_I}$$ ← è op non Hermitiano

$\propto I(\omega_{IF})$ ← segnale di uscita dell'eterodina

POVM eterodina: uso la fn caratteristica

$$P(z) = P(\mathrm{Re}\, z, \mathrm{Im}\, z) \overset{\text{regola Born}}{=} \mathrm{Tr}\left[\rho_{a_s} \Pi_z\right] \quad \forall \rho$$

$$= \int \frac{d^2\lambda}{\pi^2} \underbrace{\langle e^{\lambda \hat{z}_1^\dagger - \lambda^* \hat{z}_1} \rangle_{SI}}_{= \mathrm{Tr}[\rho_s \langle e^{\lambda \hat{z}_1^\dagger - \lambda^* \hat{z}_1} \rangle_I]} e^{\lambda^* z - \lambda z^*}$$

← anti TF in $\mathbb{C}$ della fn caratteristica

$$\chi(\lambda) \underset{\lambda \in \mathbb{C}}{=} \langle e^{\lambda \hat{z}_1^\dagger - \lambda^* \hat{z}_1} \rangle_{SI}$$

$$\Rightarrow \Pi_z = \int \frac{d^2\lambda}{\pi^2} \langle e^{\lambda \hat{z}_1^\dagger - \lambda^* \hat{z}_1} \rangle_I \, e^{\lambda^* z - z^* \lambda} =$$

$|0\rangle_I$

$$= \int \frac{d^2\lambda}{\pi^2} {}_I\langle 0| \underbrace{e^{\lambda \overbrace{(a_s^\dagger + a_I)}^{z^\dagger} - \lambda^* \overbrace{(a_s + a_I^\dagger)}^{z}}}_{\overset{[a_s, a_I] = 0}{=} e^{\lambda a_s^\dagger - \lambda^* a_s} \, e^{\lambda a_I - \lambda^* a_I^\dagger}} |0\rangle_I \, e^{\lambda^* z - z^* \lambda}$$

$e^{\lambda a_s^\dagger - \lambda^* a_s} \to D_{a_s}(\lambda)$

$$e^{\lambda a_I - \lambda^* a_I^\dagger} \overset{\text{BCH di Wh}}{=} e^{-\lambda^* a_I^\dagger} e^{\lambda a_I} e^{-\frac{|\lambda|^2}{2}}$$

$$= \int \frac{d^2\lambda}{\pi^2}\, D_{a_s}(\lambda)\, e^{\lambda^* z - z^*\lambda - \frac{|\lambda|^2}{2}}\; \underbrace{{}_I\langle 0|\, e^{-\lambda^* a_I^\dagger}\, e^{\lambda a_I} |0\rangle_I}_{=1}$$

$a_I^\dagger {}_I\langle 0| = 0$, $\quad a_I|0\rangle_I = 0$

↑ operatore sul modo a_s

$$= \int \frac{d^2\lambda'}{\pi^2}\, e^{z^*\lambda' - \lambda'^* z - \frac{|\lambda'|^2}{2}}\, \underbrace{D_{a_s}(-\lambda')}_{= D^\dagger(\lambda')} = |z\rangle\langle z|$$

$\lambda' \overset{def}{=} -\lambda$

avevamo dimostrato nella dim della identità tomografica

$$\boxed{\text{POVM eterodina è un proiettore sul coerente } |z\rangle}$$

misura congiunta di osservabili non commutanti

↳ $\Rightarrow$ rumore aggiunto

il numero z si ottiene a partire dal segnale di uscita dell'eterodina $\propto A_0 \sin(\omega_{IF} t + \delta) =$

$$= A(\cos\omega_{IF} t \sin\delta + \sin\omega_{IF} t \cos\delta)$$

$$\left.\begin{aligned} \mathrm{Re}\, z &\overset{def}{=} A \sin\delta \\ \mathrm{Im}\, z &\overset{def}{=} A \cos\delta \end{aligned}\right\} z \in \mathbb{C}$$

uscita dell'eterodina è descritta dall'op $I(\omega_{IF}) \propto a_s^\dagger + a_I = Z$

$$Z = a_s^\dagger + a_I = \hat{Z}_R + i\hat{Z}_I$$

$$Z_R = \frac{Z + Z^\dagger}{2} \qquad Z_I = -i\,\frac{Z - Z^\dagger}{2} \leftarrow \text{op auto aggiunti}$$

$$\langle Z \rangle = \langle Z_R \rangle + i \langle Z_I \rangle =$$

$$P = -i\,\frac{a-a^+}{\sqrt{2}}$$

$$= \underbrace{\left\langle \frac{a_s + a_s^+}{2} \right\rangle + \underbrace{\left\langle \frac{a_I + a_I^+}{2} \right\rangle}_{=0}}_{\langle Z_R \rangle} + i \left\langle -i\,\frac{(a_s - a_s^+)}{2} \right\rangle +$$

$$+ i \underbrace{\left\langle -i\,\frac{(a_I - a_I^+)}{2} \right\rangle}_{=0}$$

il modo a_I è nel $|0\rangle$

$$X = \frac{a+a^+}{\sqrt{2}}$$

$$= \mathrm{Re}\, z + i\, \mathrm{Im}\, z$$

$$= \frac{1}{\sqrt{2}} \langle X \rangle + i \frac{1}{\sqrt{2}} \langle P \rangle$$

$$\boxed{\begin{aligned} \langle Z_R \rangle &= \frac{1}{\sqrt{2}} \langle X \rangle \\ \langle Z_I \rangle &= \frac{1}{\sqrt{2}} \langle P \rangle \end{aligned}}$$

$$\Delta Z_R^2 = \langle Z_R^2 \rangle - \langle Z_R \rangle^2 =$$

$$\langle Z_R \rangle = \frac{\langle X \rangle}{\sqrt{2}}$$

$$= \left\langle \left(\frac{a_s^+ + a_i + a_s + a_i^+}{2} \right)^2 \right\rangle - \frac{\langle X \rangle^2}{2} =$$

$|0\rangle_I\ \rho_s$

tutti i termini che contengono un numero dispari di a_I o a_I^+ sono nulli

l'unico termine non nullo che contiene a_I, a_I^+ è $\langle a_I a_I^+ \rangle$

$$= \left\langle \frac{\left(a_s^+ a_s + a_s a_s^+ + a_s^+ a_s^+ + a_s a_s \right)}{4} + \frac{a_I a_I^+}{4} \right\rangle - \frac{\langle X \rangle^2}{2}$$

$$= \frac{\langle X^2 \rangle}{2} \qquad a_I a_I^+ = a_I^+ a_I + 1$$

$$= \frac{\langle x^2\rangle}{2} - \frac{\langle x\rangle^2}{2} + \Big\langle \underbrace{\cancel{a_I^\dagger a_I}}_{|0\rangle_2} + \frac{1}{4}\Big\rangle_I = \frac{\langle x^2\rangle}{2} - \frac{\langle x\rangle^2}{2} + \frac{1}{4}$$

$$\Delta z_I^2 = \frac{\langle p^2\rangle}{2} - \frac{\langle p\rangle^2}{2} + \frac{1}{4}$$

$$\Delta z_R^2 = \frac{\Delta x^2}{2} + \frac{1}{4}$$

rumore aggiunto

$$\Delta z_I^2 = \frac{\Delta p^2}{2} + \frac{1}{4}$$

confronto con relazioni indet Heis-Robertson:

$$\Delta x^2 \Delta p^2 \geq \frac{1}{4} \left|\langle \overbrace{[x,p]}^{i} \rangle\right|^2 = \frac{1}{4}, \iff \boxed{\frac{\Delta x^2}{2}\frac{\Delta p^2}{2} \geq \frac{1}{16}}$$

$$\Delta Z_R^2 \, \Delta Z_I^2 = \left(\frac{\Delta x^2}{2} + \frac{1}{4}\right)\left(\frac{\Delta p^2}{2} + \frac{1}{4}\right) =$$

$$\frac{\Delta^2 x \Delta^2 p}{4} + \frac{1}{16} + \frac{1}{8}\left(\Delta x^2 + \Delta p^2\right) \geq \frac{\Delta x^2 \Delta p^2}{4} + \frac{1}{16} + \frac{1}{8}\underbrace{\left(\frac{1}{4\Delta p^2} + \Delta p^2\right)}_{\frac{1+4x^4}{4x^2} \geq 1}$$

$$\Delta x^2 \geq \frac{1}{4\Delta p^2}$$

H-R

$$\geq \frac{1}{4}$$

$$\geq \frac{\Delta x^2 \Delta p^2}{4} + \frac{1}{16} + \frac{1}{8} \geq \frac{1}{4}$$

$$\boxed{\Delta Z_R^2 \, \Delta Z_I^2 \geq \frac{1}{4}} \qquad \boxed{\frac{\Delta x^2}{2}\frac{\Delta p^2}{2} \geq \frac{1}{16}}$$

$$\Delta Z_R \, \Delta Z_I \geq \frac{1}{2} \qquad \frac{\Delta x^2}{\sqrt{2}}\frac{\Delta p}{\sqrt{2}} \geq \frac{1}{4}$$

IL RUMORE dovuto alla misura congiunta si è raddoppiato

CONCLUSIONE → RIVELAZ

DIRETTA → misura dell'operatore $N = a^\dagger a$

OMODINA → " " " $X_\varphi = \int dx\, x\, |x\rangle\langle x|$

ETERODINA → " " " $\hat{Z}$ → misura congiunta di $\hat{x}$ e $\hat{p}$

CORSO di: OTTICA Q

1. Ripasso di MQ : struttura assiomatica
2. quantizz del campo elm
3. interaz radiaz materia → Ham J-C
4. Metodi algebrici → per MQ → Formule BCH
5. st quantistici radiaz
6. interenza → interometria
7. sistemi aperti
8. rivelazione

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