OTTICA Q 9/12/20

Master eq con approccio fenomenologico

$$\frac{d\rho_{SE}^I(t)}{dt} = -\frac{i}{\hbar}\left[H_{SE}^I(t), \rho_{SE}^I(t)\right]$$

$$\rho_{SE}^I(t) - \rho_{SE}^I(0) = -\frac{i}{\hbar}\int_0^t dt' \left[H_{SE}^I(t'), \rho_{SE}^I(t')\right]$$

rinomino l'Ham di sist

$$\frac{d\rho_{S}^I(t)}{dt} = \mathrm{Tr}_E\left[\cancel{-\frac{i}{\hbar}\left[H_{SE}^I(t), \rho_{SE}^I(0)\right]} - \frac{1}{\hbar^2}\int_0^t dt' \times\right.$$
$$\left.\times\left[H_{SE}^I(t), \left[H_{SE}^I(t'), \rho_{SE}^I(t')\right]\right]\right]$$

Approx di Born $\rightarrow$ $\rho_{SE}^I(t) \sim \rho_S^I(t) \otimes \rho_E^I(0)$

t in pit di interaz

l'evoluz dello stato è governata da H_{SE}^I

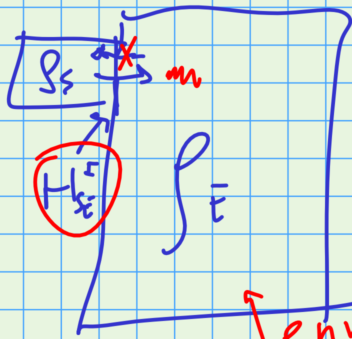
interaz debole + environment grande

$$\rho_E^I(t) \sim \rho_E^I(0) = \rho_E^S(0)$$

$$\frac{d\rho_{SE}^I(t)}{dt} = -\frac{1}{\hbar^2}\int_0^t dt'\, \mathrm{Tr}_E\left[H_{SE}^I(t), \left[H_{SE}^I(t'), \rho_{SE}^I(t')\otimes\rho_E^I(0)\right]\right]$$

lo st al tempo t dipende dagli stati ai tempi precedenti

APPROX di MARKOV $\rightarrow$ in pitt di interaz la variaz dello st dipende da H_{SE}^S, se i gdl dell'E hanno dinamica veloce



variaz di ρ_S è indip dai tempi precedenti se $\rho_E \to$ rilassa rapidamente

environment veloce: il suo stato si riposiziona rapidamente

$$\frac{d\rho_S^I}{dt} = -\frac{1}{\hbar^2}\int_0^t dt'\, Tr_E\left[H_{SE}^I(t), \left[H_{SE}^I(t'), \rho_S(\cancel{t'}\ t)\otimes\rho_E(0)\right]\right]$$

approx Markov

Approx di Markov $\to$ usando le fn di correlaz dell'ambiente.

op di sist, op di environment

$$H_{SE} = \hbar \sum_k S_k \otimes E_k$$

$\forall$ stato di uno sp di Hilbert bipartito si può scrivere con decomposiz Schmidt

$$|\psi_{AB}\rangle = \sum_i \lambda_i\, |i\rangle_A\, |i\rangle_B$$

Base di $\mathcal{H}_A$ — base di $\mathcal{H}_B$

decomposiz Schmidt di H_{SE} nello sp di Hilbert di operatori

$$H_{SE}^I(t) = \hbar \sum_k S_k^I(t) \otimes E_k^I(t)$$

$$S_k^I = e^{\frac{i}{\hbar}H_S t}\, S_k\, e^{-\frac{i}{\hbar}H_S t}$$

No Markov

$$E_k^I = e^{\frac{i}{\hbar}H_E t}\, E_k\, e^{-\frac{i}{\hbar}H_E t}$$

$$\frac{d\rho^I}{dt} = -\frac{1}{\hbar^2}\int_0^t dt'\, Tr_E\Big[H_{SE}^I(t)\left(H_{SE}^I(t')\,\rho^I(t') - \rho^I(t')\,H_{SE}^I(t')\right)$$

$$-\left(\ \text{"}\qquad\text{"}\qquad\text{"}\ \right)H_{SE}^I(t)$$

$$= -\int_0^t dt' \sum_{jk} \Big(S_j(t)\, S_k(t')\, \rho(t') - S_j(t)\, \rho(t') S_k(t') \Big) \Gamma_{jk}(t,t')$$

$$\Gamma_{jk}(t,t') \stackrel{def}{=} \mathrm{Tr}_E \left[E_j(t)\, E_k(t')\, \rho_E \right]$$

scambio j, k come variabili di somma

$$- \Big(S_k(t')\, \rho(t') S_j(t) - \rho\, S_k(t')\, S_j(t) \Big) \Gamma_{kj}(t,t')$$

$\Gamma(t,t') \sim \delta(t-t')$ fn di correlaz delta Dirac

rappresentano Environment veloce

$$= -\sum_{jk} \Gamma'_{jk} \left(S_j^I S_k^I \rho^I + \rho^I S_j^I S_k^I - 2 S_j^I \rho^I S_k^I \right) = \frac{d\rho_s^I}{dt}$$

$\int dt'$ va via $\quad \rho(t') \to \rho(t) \qquad \Gamma_{jk} = \Gamma_{kj}$

passo alla pittura Schroedinger

$$\left.\begin{aligned} \rho^I &= U_1^+ \left(U \rho_0 U^+ \right) U_1 \\ \rho^s &= U \rho_0 U^+ \end{aligned}\right\} \Rightarrow \rho^s = U_1\, \rho\, U_1^+$$

$$U_1 = e^{-\frac{i}{\hbar}(H_s + H_E)t}$$

$$S^I = U_1^+ S^s U_1$$

$$S^s = U_1 S^I U_1^+$$

$$\frac{d}{dt}\Big(\underbrace{U_1^+ \rho^s U_1}_{\rho^I} \Big) = -\sum_{jk} \Gamma'_{jk} \Big(\underbrace{U_1^+ S_j^s U_1 U_1^+ S_k^s U_1 U_1^+ \rho U_1}_{S_j^I S_k^I \rho^I} + U_1^+ \rho S_j S_k U_1 - 2 U_1^+ S_j \rho S_k U_1 \Big)$$

$$\frac{dU_1^+}{dt} \rho U_1 + U_1^+ \frac{d\rho^s}{dt} U_1 + U_1^+ \rho \frac{dU_1}{dt}$$

$$\frac{dU_1^+}{dt} = +\frac{i}{\hbar}(H_s + H_E) U_1^+ \qquad \frac{dU_1}{dt} = -\frac{i}{\hbar}(H_s + H_E) U_1$$

$$\cancel{U_1^+} \frac{d\rho^s}{dt} \cancel{U_1} = \cancel{U_1^+} \left[-\frac{i}{\hbar} \left[H_s + \cancel{H_E}, \rho_s \right] - \sum_{jk} \Gamma_{jk} \left(S_j S_k \rho + \rho S_j S_k - 2 S_j \rho S_k \right) \right] \cancel{U_1}$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_s, \rho] - \sum_{jk} \Gamma_{jk} \left(S_j S_k \rho + \rho S_j S_k - 2 S_j \rho S_k \right)$$

m. eq in forma di Kossakowski

per avere Lindblad $\rightarrow$ basta diagonalizzare Γ_{jk}

le due derivazioni date della M equation $\begin{cases} \to \text{q dyn semig} \\ \to \text{fenomenol} \end{cases}$ sono equivalenti

M eq in pittura di Heisenberg

$\rightarrow$ trucco dell'invar perm ciclica tr:

$$\frac{d\rho}{dt} = \mathcal{L}[\rho] \Rightarrow \frac{d\langle X\rangle}{dt} = \mathrm{Tr}\left[X \mathcal{L}[\rho]\right] =$$

$$\overset{def}{=} \mathrm{Tr}\left[\mathcal{L}^{\vee}[X]\rho\right]$$

$\uparrow$ Liouvilliano duale

$$\mathcal{L}[\rho] = \sum_m C_m \rho C_m^{+} - \frac{1}{2}\left\{C_m^{+} C_m \rho + \rho C_m^{+} C_m\right\}$$

$$\mathrm{Tr}\left[X \mathcal{L}[\rho]\right] = \mathrm{Tr}\left[\sum_m X\left(C_m \rho C_m^{+} - \frac{1}{2}\left(C_m^{+} C_m \rho + \rho C_m^{+} C_m\right)\right)\right]$$

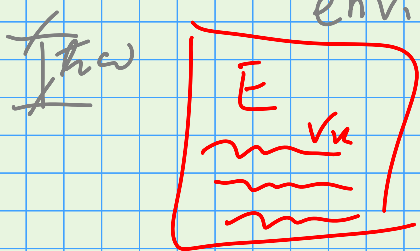
$$= \mathrm{Tr}\left[\sum_m C_m^{+} X C_m \rho - \frac{1}{2}\left(X C_m^{+} C_m \rho + C_m^{+} C_m X \rho\right)\right]$$

$$\Rightarrow \mathcal{L}^{\vee}[X] = \sum_m C_m^{+} X C_m - \frac{1}{2}\left(C_m^{+} C_m X + X C_m^{+} C_m\right)$$

quando abbiamo ricavato la m. eq dalla forma fenomenologica non c'erano ; dagga $\rightarrow$ S_j sono op autoaggiunti

$$H_{SE} = \hbar \sum_k \underbrace{S_k}_{\text{op autoag}} \otimes E_k$$

ESEMPIO di M EQUATION $\rightarrow$ atomo a 2 livelli in interaz con $\underbrace{\text{radiaz termica a molti modi}}_{\text{environment}}$ $\rightarrow$ emissione spontanea



H interaz $\rightarrow$ H di Jaynes Cummings

RWA pittura di interaz:

$$H^I_{SE}(t) = \hbar \sum_k g_{\vec{k}} \left[a^\dagger_{\vec{k}} \sigma_- e^{-i(\omega-\nu_k)t} + a_k \sigma_+ e^{+i(\omega-\nu_k)t} \right]$$

JC

radiaz è in st termico $\rho_E = \bigotimes_{\vec{k}} \left(1 - e^{-\beta_k}\right) e^{-\beta_k a^\dagger_{\vec{k}} a_{\vec{k}}}$

$$\beta_k \overset{\text{def}}{=} \frac{\hbar \nu_k}{k_B T}$$

st termico è diagonale nella base Fock

$\Rightarrow$ $\langle a_k \rangle = 0$ $\qquad \langle a_k \rangle = \mathrm{Tr}\left[a_k \rho \right] = 0$ $\qquad \rho = \sum_n \alpha_n |n\rangle\langle n|$

$\langle a^\dagger_k \rangle = 0$

$\langle a^\dagger a^\dagger \rangle = 0 \qquad \langle a_k a_{k'} \rangle = 0$

Sopravvivono solo $\langle a^\dagger_k a_k \rangle$, $\langle a_k a^\dagger_k \rangle$

M eq di Born-Markov:

$$\frac{d\rho_s^I}{dt} = \cancel{-\frac{i}{\hbar} \mathrm{Tr}_i \left[H^I_{SE}(t), \rho^I_s(t) \otimes \rho_E(0) \right]} - \frac{1}{\hbar^2} \mathrm{Tr}_E \int_0^t dt' \left[H^I_{SE}(t), \left[H^I_{SE}(t'), \rho^I_s(t) \otimes \rho_E \right] \right] =$$

$a^\dagger \sigma_- + a \sigma_+$ $\qquad \langle a \rangle = 0$ $\qquad \langle a^\dagger_k \rangle = 0$

sistema = atomo $\qquad E$ = radiaz

$$
\left|\begin{array}{lll}
a' = a_{k'} & e' = e^{-i(\omega-\nu_{k'})t} & \rho = \rho_s^I(t)\otimes\rho_E \\
a = a_k & e = e^{-i(\omega-\nu_k)t} &
\end{array}\right|
$$

$$
\sum_{kk'} g_k g_{k'} \left[a\sigma_+ e^* + a^\dagger \sigma_- e \,,\, \left[a'\sigma_+ e'^* + a'^\dagger \sigma_- e' , \rho \right]\right] =
$$

$$
= \sum_{kk'} g_k g_{k'} \left[a\sigma_+ e^* + a^\dagger \sigma_- e \,,\, \left(a'\sigma_+ e'^*\rho + a'^\dagger\sigma_- e'\rho - \rho a'\sigma_+ e'^* - \rho a'^\dagger \sigma_- e' \right)\right] =
$$

mantengo SOLO i termini che contengono $a\, a^\dagger$ — i termini con a^2 e $a^{\dagger 2}$ non contribuiscono Tr_E

$$
\to \sum_{kk'} g_k g_{k'} \Big(a\sigma_+ e^* a'^\dagger \sigma_- e' \rho - a\sigma_+ e^* \rho a'^\dagger \sigma_- e' + a^\dagger \sigma_- e\, a'\sigma_+ e'^* \rho - a^\dagger \sigma_- e\, \rho a' \sigma_+ e'^*
$$

$$
- \left[a'^\dagger \sigma_- e' \rho a \sigma_+ e^* - \rho a'^\dagger \sigma_- e' a \sigma_+ e^* + a'\sigma_+ e'^* \rho a^\dagger \sigma_- e - \rho a' \sigma_+ e'^* a^\dagger \sigma_- e \right] \Big) =
$$

Tr_E

$$
= \sum_{kk'} g_k g_{k'} \Big(\underbrace{\langle a a'^\dagger \rangle}_{\delta_{kk'}} \sigma_+\sigma_- \rho_s e^* e' - \overbrace{Tr_E[a\rho_E a'^\dagger]}^{\langle a'^\dagger a\rangle} \sigma_+ \rho_s \sigma_- e^* e'
$$

$$
+ \overbrace{\langle a^\dagger a' \rangle}^{\delta_{kk'}} \sigma_-\sigma_+ \rho\, e e'^* - \langle a' a^\dagger \rangle \sigma_- \rho \sigma_+ e e'^*
$$

$$
- \Big[\overbrace{\langle a a'^\dagger \rangle} \sigma_- \rho \sigma_+ e' e^* - \langle a^\dagger a \rangle \rho_s \sigma_- \sigma_+ +
$$

$$
+ \langle a^\dagger a' \rangle \sigma_+ \rho \sigma_- e'^* \underset{n+1}{e} - \rho \sigma_+ \sigma_- \langle a' a^\dagger \rangle \Big]
$$

$$
\frac{d\rho_s^I}{dt} = - \int_0^t dt' \sum_k g_k^2 \Big[\Big(\underbrace{\langle a a^\dagger \rangle}_{a^\dagger a + 1} \sigma_+ \sigma_- \rho - \langle a^\dagger a \rangle \sigma_+ \rho \sigma_-
$$

$$+\langle a^+a\rangle \sigma_-\sigma_+\rho \;-\; \langle a a^+\rangle \sigma_-\rho\sigma_+\Big) e^{-i(\omega-\nu_k)(t-t')}$$

$\langle aa^+\rangle = a^+a+1$

$$-\Big[\langle aa^+\rangle \sigma_-\rho\sigma_+ - \langle a^+a\rangle \rho\sigma_-\sigma_+ + \langle a^+a\rangle \sigma_+\rho\sigma_- - \langle aa^+\rangle \rho\sigma_+\sigma_-\Big] e^{+i(\omega-\nu_k)(t-t')} =$$

Γ contiene $\int_k \int dt$

$$= -\frac{\Gamma}{2}\Big[(\bar n+1)\,\sigma_+\sigma_-\rho - \bar n\,\sigma_+\rho\sigma_- + \bar n\,\sigma_-\sigma_+\rho$$

$\bar n$ = n. medio di fotoni del termico $\langle a^+a\rangle = \bar n$

$$-(\bar n+1)\sigma_-\rho\sigma_+ - \Big((\bar n+1)\sigma_-\rho\sigma_+ - \bar n\,\rho\sigma_-\sigma_+ + \bar n\,\sigma_+\rho\sigma_-$$

$$-(\bar n+1)\rho\sigma_+\sigma_-\Big)\Big] =$$

$$= \frac{\Gamma}{2}\Big(\bar n\,(\sigma_+\rho\sigma_- - \sigma_-\sigma_+\rho - \rho\sigma_-\sigma_+ + \sigma_+\rho\sigma_-) +$$

$$2\bar n\left(\sigma_+\rho\sigma_- - \frac{1}{2}(\sigma_-\sigma_+\rho + \sigma_-\sigma_+\rho)\right)$$

$$+(\bar n+1)\big[\sigma_+\sigma_-\rho - \sigma_-\rho\sigma_+ - \sigma_-\rho\sigma_+ + \rho\sigma_+\sigma_-\big]$$

$$-2(\bar n+1)\left(\sigma_-\rho\sigma_+ - \frac{1}{2}(\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-)\right)$$

$$\frac{d\rho_s^I}{dt} = \Gamma\Big[\bar n\, D[\sigma_+]\rho + (\bar n+1)\, D[\sigma_-]\rho\Big]$$

m eq in forma di Lindblad

termine di eccitaz dell'atomo: atomo transisce dal fondam all'eccitato $\sigma_+\rho\sigma_-$
il termine rappresenta l'atomo che assorbe i fotoni termici

termine di diseccitaz dell'atomo $\sigma_-\rho\sigma_+$ fa transire l'atomo al liv fondam

⇒ è un termine che rappresenta l'atomo che perde energia emettendo fotoni nel vuoto

se $\bar{n}=0$ sopravvive il secondo termine

⇒ l'atomo emette un fotone solo per interaz con dei modi nel vuoto $(\bar{n}=0)$ → EMISSIONE SPONTANEA