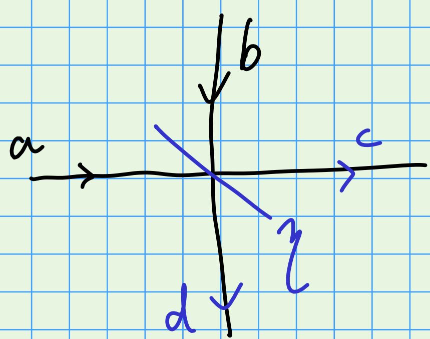
# OTTICA Q 1/12/20

INTERFERENZA QUANTISTICA → interferometria

BEAM SPLITTER → specchio semitrasparente

fa passare una fraz $\eta$ dell'intensità
e riflette una fraz $1-\eta$  $\eta \in [0,1]$




in pitt di Heisenberg

$$c = \sqrt{\eta}\, a + \sqrt{1-\eta}\, b$$

$\eta$ per l'intensità $\Rightarrow$ $\sqrt{\eta}$ per l'ampiezza

$$d = \sqrt{\eta}\, b - \sqrt{1-\eta}\, a \qquad [c, d^+] = 0$$

c, d non sarebbero modi

Alternativamente:

$H \propto a b^+ - a^+ b$

$$\boxed{\begin{aligned} U &\overset{+}{=} \exp\left[-\operatorname{arctg}\left(\sqrt{\eta^{-1}-1}\right)\left(a b^+ - a^+ b\right)\right] = \\ &\underset{BCH}{\overset{\downarrow}{=}} e^{a^+ b \sqrt{\eta^{-1}-1}}\ \eta^{-\frac{1}{2}(a^+ a - b^+ b)}\ e^{-a b^+ \sqrt{\eta^{-1}-1}} \equiv \\ &\quad e^{-a b^+ \sqrt{\eta^{-1}-1}}\ \eta^{+\frac{1}{2}(a^+ a - b^+ b)}\ e^{+a^+ b \sqrt{\eta^{-1}-1}} \end{aligned}}$$

c e d sono modi

$$[c, c^+] = [\sqrt{\eta}\, a + \sqrt{1-\eta}\, b, \sqrt{\eta}\, a^+ + \sqrt{1-\eta}\, b^+] =$$
$$= \eta [a, a^+] + (1-\eta)[b, b^+] = \eta + 1 - \eta = 1$$

$[a, b^+] = 0$

$$[d, d^+] = 1$$

$$[c, d^+] = [\sqrt{\eta}\, a + \sqrt{1-\eta}\, b, \sqrt{\eta}\, b^+ - \sqrt{1-\eta}\, a^+] = -\sqrt{\eta(1-\eta)}\, \overbrace{[a, a^+]}^{1}$$
$$+ \sqrt{\eta(1-\eta)}\, \overbrace{[b, b^+]}^{1} = 0$$

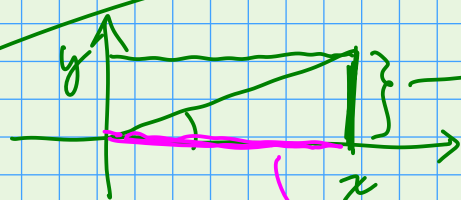
$$c \overset{?}{=} U^\dagger a U = e^{\gamma(ab^\dagger - a^\dagger b)}\, a\, e^{-\gamma(ab^\dagger - a^\dagger b)} =$$

Heisenberg $\qquad \gamma \overset{def}{=} \operatorname{arctg}\sqrt{\eta^{-1}-1}$

$$e^{A} B e^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \dots$$

$$= a + \gamma\underbrace{[\overbrace{ab^\dagger - a^\dagger b}^{A}, \overbrace{a}^{B}]}_{b} + \frac{\gamma^2}{2!}\underbrace{[ab^\dagger - a^\dagger b, b]}_{-a} + \frac{\gamma^3}{3!}\underbrace{[ab^\dagger - a^\dagger b, -a]}_{-b}$$

$$+ \dots = a\underbrace{\sum_{n=0}^{\infty}\frac{\gamma^{2n}}{(2n)!}(-1)^n}_{\cos\gamma} + b\underbrace{\sum_{n=0}^{\infty}\frac{\gamma^{2n+1}}{(2n+1)!}(-1)^n}_{\sin\gamma} =$$




$$\sin \operatorname{arctg}\frac{y}{x} = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos \operatorname{arctg}\frac{y}{x} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\gamma = \operatorname{arctg}\sqrt{\eta^{-1}-1} = \operatorname{arctg}\sqrt{\frac{1-\eta}{\eta}} \Rightarrow \frac{x}{\sqrt{x^2+y^2}} = \sqrt{\eta}$$

$$\frac{y}{\sqrt{x^2+y^2}} = \sqrt{1-\eta}$$

$$= a\,\overbrace{\cos \operatorname{arctg}\sqrt{\frac{1-\eta}{\eta}}}^{\sqrt{\eta}} + b\,\overbrace{\sin \operatorname{arctg}\sqrt{\frac{1-\eta}{\eta}}}^{\sqrt{1-\eta}}$$

$$= \boxed{a\sqrt{\eta} + b\sqrt{1-\eta} = c}$$

$$U^\dagger b U = d$$

BCH : Formula di su(2)

$$J_+ = a^\dagger b = (J_-)^\dagger \; ; \; J_z = \frac{1}{2}(a^\dagger a - b^\dagger b)$$

$$[J_+, J_-] = [a^\dagger b, a b^\dagger] = \underbrace{2 J_z = a^\dagger a - b^\dagger b}$$

$$[J_z, J_\pm] = \pm J_\pm \quad \leftarrow [a, a^\dagger] = 1 \quad [b, b^\dagger] = 1 \quad [a, b^\dagger] = 0$$

$$e^{-\gamma(\underset{J_+}{a b^\dagger} - \underset{J_-}{a^\dagger b})} \underset{\text{BCH } su(2)}{=} e^{\zeta a^\dagger b} \underbrace{(\cos|\gamma|)^{-2 J_z}} e^{-\zeta^* a b^\dagger} =$$

$$\cos|\gamma| = \sqrt{\eta} \qquad \zeta = \frac{\gamma}{|\gamma|} \, tg|\gamma| = \sqrt{\eta^{-1} - 1} \qquad \gamma = \mathrm{arctg}\sqrt{\eta^{-1}-1}$$

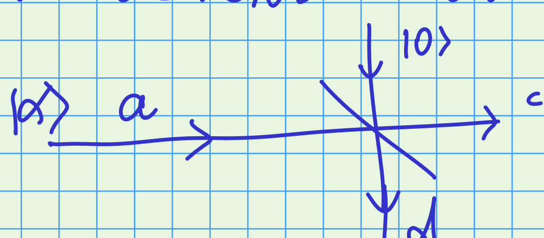
$$= e^{\sqrt{\eta^{-1}-1}\, a^\dagger b} \; \eta^{b^\dagger b - a^\dagger a} \; e^{-\sqrt{\eta^{-1}-1}\, a b^\dagger}$$

la seconda BCH si ottiene da questa semplic scambiando i modi $a \to b$, $b \to a$

$$\underline{U^\dagger a U = \sqrt{\eta}\, a + \sqrt{1-\eta}\, b = c}$$

$$\boxed{U a U^\dagger = \sqrt{\eta}\, a - \sqrt{1-\eta}\, b} \leftarrow \text{conto di prima con segno di } \gamma \text{ invertito}$$

• EVOLUZIONE di UN COERENTE (laser che illumina beam splitter con nulla che entra dall'altra porta)




$$U|\alpha\rangle_a |0\rangle_b = U \underbrace{D_a(\alpha)|0\rangle_a}_{|\alpha\rangle_a} |0\rangle_b = \qquad D_a(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

$$= U D_a(\alpha) \overbrace{U^\dagger U}^{\mathbb{1}} |0\rangle_a |0\rangle_b =$$

$$U|0\rangle_a |0\rangle_b = |0\rangle|0\rangle \quad (\text{BCH})$$

$$U|0\rangle|0\rangle = e^{-\zeta a^\dagger b}\,\eta^{b^\dagger b - a^\dagger a}\,e^{-\zeta^* a b^\dagger}|0\rangle|0\rangle$$

$$= |0\rangle|0\rangle$$

$$= \underbrace{U e^{\overset{\sqrt{\eta}a^\dagger - \sqrt{1-\eta}b^\dagger}{\alpha a^\dagger} - \overset{\sqrt{\eta}a - \sqrt{1-\eta}b}{\alpha^* a}} U^\dagger}\, |0\rangle|0\rangle =$$

$$U a U^\dagger = \sqrt{\eta}\,a - \sqrt{1-\eta}\,b$$

$$U a^\dagger U^\dagger = \sqrt{\eta}\,a^\dagger - \sqrt{1-\eta}\,b^\dagger$$

$$= e^{\alpha(\sqrt{\eta}\,a^\dagger - \sqrt{1-\eta}\,b^\dagger) - \alpha^*(\sqrt{\eta}\,a - \sqrt{1-\eta}\,b)}\,|0\rangle|0\rangle$$

$$= \underbrace{e^{\alpha\sqrt{\eta}\,a^\dagger - \alpha^*\sqrt{\eta}\,a}}_{D_a(\sqrt{\eta}\,\alpha)}\ \underbrace{e^{-\alpha\sqrt{1-\eta}\,b^\dagger + \alpha^*\sqrt{1-\eta}\,b}}_{D_b(-\alpha\sqrt{1-\eta})}\,|0\rangle_a|0\rangle_b =$$

a, b commutano

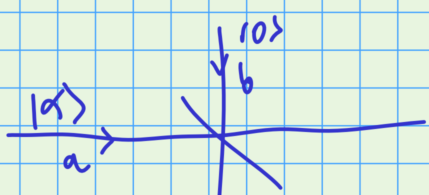
$$= D_a(\sqrt{\eta}\,\alpha)|0\rangle_a \otimes D_b(-\sqrt{1-\eta}\,\alpha)|0\rangle_b =$$

$$= |\sqrt{\eta}\,\alpha\rangle_a\,|-\sqrt{1-\eta}\,\alpha\rangle_b$$

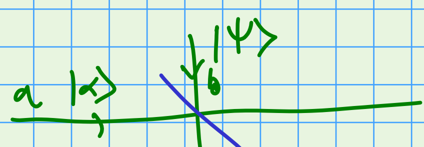
Coerente in ingresso viene diviso in due coerenti

→ in parte è riflesso e in parte è trasmesso

OSSERVAZIONE $D_a(\alpha)$ è in generale l'evoluz che si ha quando illumino un beam splitter con un coerente

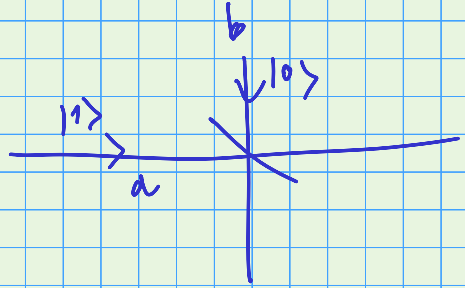



$$= D_a(\alpha)\,|0\rangle_a|0\rangle_b$$




$$= D_a(\alpha)|0\rangle_a\,|\psi\rangle_b$$

## • EVOLUZ di un SINGOLO FOTONE




$$U|1\rangle_a|0\rangle_b = U a^\dagger |0\rangle_a |0\rangle_b =$$

$$= U a^\dagger \underbrace{U^\dagger U}_{1} \overbrace{|0\rangle_a |0\rangle_b}^{|0\rangle|0\rangle} =$$

$$= \underbrace{U a^\dagger U^\dagger}_{\sqrt{\eta}\, a^\dagger - \sqrt{1-\eta}\, b^\dagger} |0\rangle|0\rangle = \left(\sqrt{\eta}\, \overbrace{a^\dagger}^{=|10\rangle} - \sqrt{1-\eta}\, \underbrace{b^\dagger}_{|01\rangle}\right)|00\rangle =$$

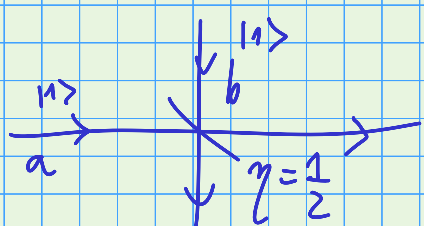
$$\boxed{\begin{aligned} &= \sqrt{\eta}\, |1\rangle_a |0\rangle_b - \sqrt{1-\eta}\, |0\rangle_a |1\rangle_b \\ &= U|1\rangle|0\rangle \end{aligned}}$$

con ampiezza di prob $\sqrt{\eta}$ è trasmesso
con ampiezza $-\sqrt{1-\eta}$ è riflesso

Ogni fotone è trasmesso con prob $\eta$
riflesso $1-\eta$

$$U|0\rangle|1\rangle = \sqrt{\eta}\, |01\rangle + \sqrt{1-\eta}\, |10\rangle$$


## • INTERFEROMETRO di HONG - OU - MANDEL




$$U|1\rangle|1\rangle$$

ci aspetteremmo che ciascun fotone è trasmesso con $p = \frac{1}{2}$ e riflesso con $p = \frac{1}{2}$ $\Rightarrow$ ci aspetteremmo che con $p = \frac{1}{4}$ → entrambi sono in c

$p = \frac{1}{4}$ → " " d

$p = \frac{1}{2}$ → escono ciascuno da una porta

↓

NO!

C'è un'interferenza quantistica dovuta alla natura quantistica della luce ⇒ HOM dimostra l'aspetto quantistico della radiazione

$$U|1\rangle_a|1\rangle_b = U a^\dagger b^\dagger |00\rangle = U a^\dagger U^\dagger U b^\dagger U^\dagger \underbrace{U|00\rangle}_{|00\rangle}$$

$$= \left(\sqrt{\eta}\, a^\dagger - \sqrt{1-\eta}\, b^\dagger\right)\left(\sqrt{\eta}\, b^\dagger + \sqrt{1-\eta}\, a^\dagger\right)|00\rangle =$$

$$U a^\dagger U^\dagger = \sqrt{\eta}\, a^\dagger - \sqrt{1-\eta}\, b^\dagger$$

$$U b^\dagger U^\dagger = \sqrt{\eta}\, b^\dagger + \sqrt{1-\eta}\, a^\dagger$$

vengono dai commutatori

$$= \left(\eta\, a^\dagger b^\dagger + \sqrt{\eta(1-\eta)}\, a^{\dagger 2} - \sqrt{\eta(1-\eta)}\, b^{\dagger 2} - (1-\eta)\, b^\dagger a^\dagger\right)|00\rangle$$

$$= \cancel{\eta\,|1\rangle|1\rangle} + \sqrt{\eta(1-\eta)}\,\sqrt{2}\,|20\rangle - \sqrt{\eta(1-\eta)}\,|02\rangle\sqrt{2}\ \cancel{-(1-\eta)|11\rangle}$$

interferenza quantistica

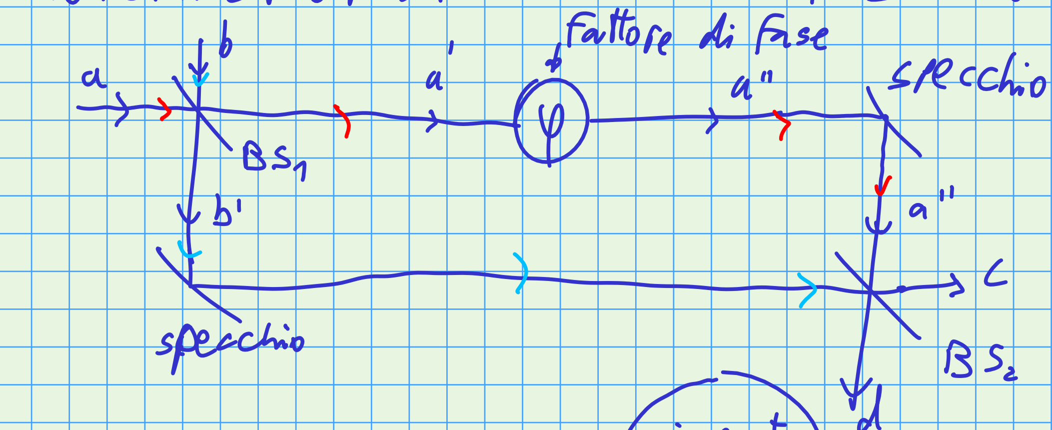
$$a^{\dagger 2}|0\rangle = \sqrt{2}\,|2\rangle$$

$$\underset{\eta=\frac{1}{2}}{=} \frac{1}{\sqrt{2}}\left(|20\rangle - |02\rangle\right)$$

entrambi i fotoni escono nel modo a

entrambi escono nel modo b

## • INTERFEROMETRO di MACH-ZEHNDER




fattore di fase $U = e^{i\varphi a^\dagger a}$

$H \propto a^\dagger a \longrightarrow$ Hamilt del campo elm libero

$$H = \hbar\omega\left(a^\dagger a + \tfrac{1}{2}\right)$$

fattore di fase $\rightarrow$ un ramo dell'interferometro è più lungo dell'altro $\varphi \propto$ differenz di tempi di viaggi nei due rami

$$U_{TOT} |1\rangle_a |0\rangle_b = \overbrace{U_{BS_2} \left( e^{i\varphi a^\dagger a}_{a} \otimes \mathbb{1}_b \right) U_{BS_1}}^{U_{TOT}} |10\rangle =$$

$$U_{BS_1}|10\rangle = \sqrt{\eta}|10\rangle - \sqrt{1-\eta}|01\rangle$$

$$= U_{BS_2} \left( e^{i\varphi a^\dagger a} \otimes \mathbb{1} \right) \left( \sqrt{\eta}\,|10\rangle - \sqrt{1-\eta}\,|01\rangle \right) =$$

$$= U_{BS_2} \left( \sqrt{\eta}\, e^{i\varphi} |10\rangle - \sqrt{1-\eta}\, |01\rangle \right) =$$

$$e^{i\varphi a^\dagger a} |10\rangle = e^{i\varphi} |10\rangle$$

$$e^{i\varphi a^\dagger a} |01\rangle = |01\rangle$$

$$= \sqrt{\eta}\, e^{i\varphi} \left( \sqrt{\eta}\, |10\rangle + \sqrt{1-\eta}\, |01\rangle \right) - \sqrt{1-\eta} \left( \sqrt{\eta}\, |01\rangle - \sqrt{1-\eta}\, |10\rangle \right)$$

$BS_2$ ha i modi invertiti rispetto al $BS_1$ $\quad a \rightarrow b$, $b \rightarrow a$

$$U_{BS_2} |10\rangle = \sqrt{\eta}\, |10\rangle + \sqrt{1-\eta}\, |01\rangle$$

$$U_{BS_2} |01\rangle = \sqrt{\eta}\, |01\rangle - \sqrt{1-\eta}\, |10\rangle$$

$$= e^{i\varphi} \left( \eta\, |10\rangle + \sqrt{\eta(1-\eta)}\, |01\rangle \right) - \sqrt{\eta(1-\eta)}\, |01\rangle + (1-\eta)\, |10\rangle$$

$$= \left( \frac{e^{i\varphi} + 1}{2} \right) |10\rangle + \left( \frac{e^{i\varphi} - 1}{2} \right) |01\rangle =$$

$\eta = \frac{1}{2}$

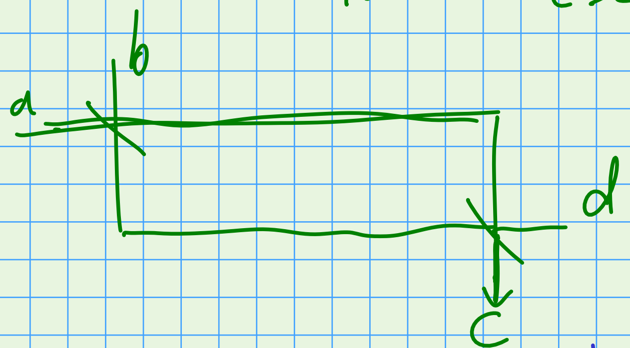
$$= e^{i\frac{\varphi}{2}} \left[ \frac{e^{i\frac{\varphi}{2}} + e^{-i\frac{\varphi}{2}}}{2} |10\rangle + \frac{e^{i\frac{\varphi}{2}} - e^{-i\frac{\varphi}{2}}}{2} |01\rangle \right] =$$

$$= e^{i\frac{\varphi}{2}} \left[ \left( \cos\frac{\varphi}{2} \right) |10\rangle + i \left( \sin\frac{\varphi}{2} \right) |01\rangle \right]$$

il fotone esce dal modo c con prob $\cos^2\frac{\varphi}{2}$

d " " $\sin^2\frac{\varphi}{2}$

SICURAM il fotone esce da c se $\varphi=0$



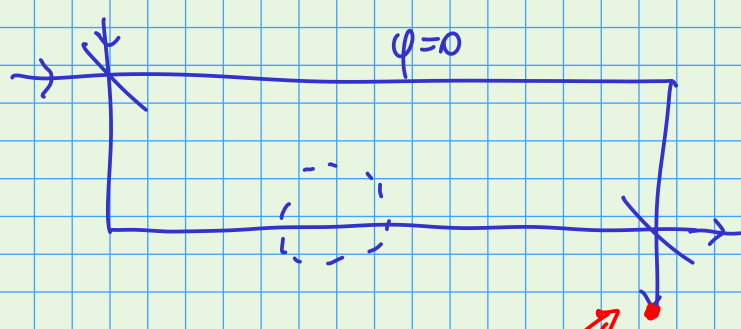

Quantum seeing in the dark

(interaction free measureament → Elitzur - Vaidman)

→ c'è una bomba sensibilissima: basta che sia colpita da un fotone ed esplode

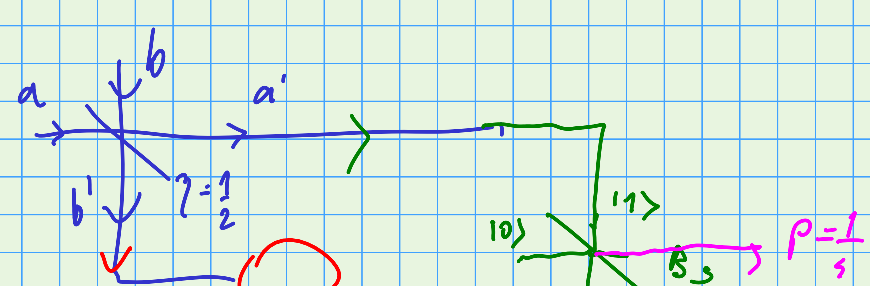
come fare a vedere se c'è la bomba?

metto un M.Z ($\varphi=0$) nella posiz dove ci potrebbe essere la bomba




se bomba non c'è ⇒ sicuramente il fotone esce da c

se bomba c'è




dopo il 1° BS

il fotone è in a' con $P=\frac{1}{2}$ e in b' con prob $\frac{1}{2}$

se è in $b'$ $\Rightarrow$ la bomba scoppia $\rightarrow$ sono sfortunato con $p = \frac{1}{2}$

se è in $a'$ (con $p = \frac{1}{2}$) $\Rightarrow$ con $p = \frac{1}{2}$ è trasmesso ed esce in c, ma con $p = \frac{1}{2}$ è riflesso ed esce in d!

$P_{TOT} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ esce da d

se vedo che il fotone esce da d $\Rightarrow$ SICURAMENTE LA BOMBA C'È, ma non è scoppiata.

$\underbrace{\boxed{\text{Funziona con } p = \frac{1}{4}}}_{\text{con } p = \frac{1}{2} \text{ la bomba scoppia}}$ $\longrightarrow$ esistono trucchi per aumentare questa prob