OTTICA Q 24/11/20

rappres differenziali delle mappe operatoriali

$$D_s(a\,\cdot) = \alpha + \frac{1-s}{2}\frac{\partial}{\partial\alpha^*}$$

$$D_s(a^\dagger\,\cdot) = \alpha - \frac{1+s}{2}\frac{\partial}{\partial\alpha}$$

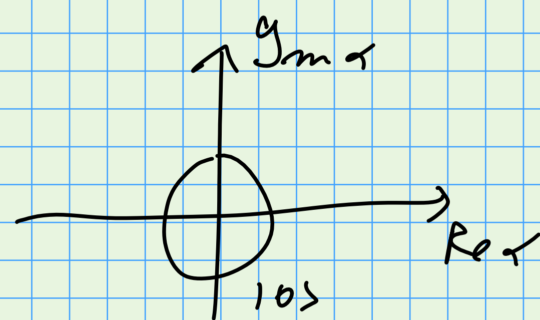
$$D_s(\cdot\, a) = \alpha - \frac{1+s}{2}\frac{\partial}{\partial\alpha^*}$$

$$D_s(\cdot\, a^\dagger) = \alpha^* + \frac{1-s}{2}\frac{\partial}{\partial\alpha}$$

Fn di Wigner dello st coerente:

$$W(\alpha, |\beta\rangle\langle\beta|) = \frac{2}{\pi} e^{-2|\alpha-\beta|^2}$$

$$\Rightarrow W(\alpha, |0\rangle\langle 0|) = \frac{2}{\pi} e^{-2|\alpha|^2}$$



$$W(\alpha, |\beta\rangle\langle\beta|) = \int\frac{d^2\lambda}{\pi^2} e^{\alpha\lambda^* - \alpha^*\lambda} \langle\beta| D(\lambda)|\beta\rangle =$$

$$|\beta+\lambda\rangle e^{\frac{1}{2}(\lambda\beta^* - \beta\lambda^*)}$$

$$\langle\beta|\beta+\lambda\rangle = e^{-\frac{|\lambda|^2}{2}} e^{\frac{1}{2}(\lambda\beta^* - \lambda^*\beta)}$$

$$= \int\frac{d^2\lambda}{\pi^2} e^{\lambda^*(\alpha-\beta) - \lambda(\alpha^*-\beta^*)} e^{-\frac{|\lambda|^2}{2}} = \frac{2}{\pi} e^{-2|\alpha-\beta|^2}$$

$$\int d^2\lambda\, e^{\alpha\lambda^* - \lambda\alpha^*} e^{-\frac{|\lambda|^2}{\sigma^2}} = e^{-\sigma^2|\alpha|^2}$$

STATO di FOCK, $|n\rangle$

è autost dell'Hamilt di singolo modo

$a^\dagger a |n\rangle = n|n\rangle$

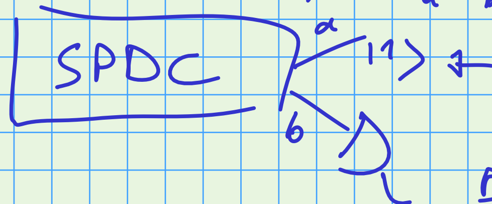
fr di Wigner → polinomi di Laguerre

$|0\rangle$ è st coerente e st di Fock

tutti gli altri non sono st a minima indet

Trucco per creare st di Fock $|1\rangle$

$$|\text{twin beam}\rangle \sim |00\rangle_{ab} + \gamma\, |11\rangle_{ab}$$



← st Fock in a condiz ad avere click nel modo b

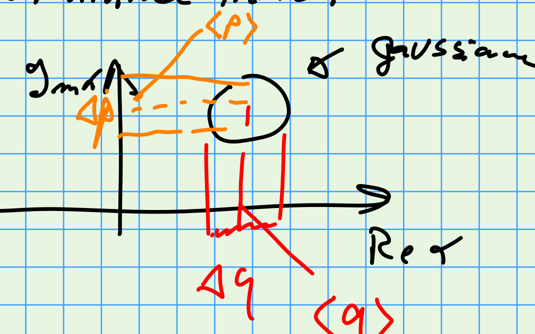
Detector → se fa click vuol dire che c'è un fotone

STATO SQUEEZED

abbiamo visto st coerenti sono a minima indet

$\Delta p \, \Delta q = \frac{1}{2}$ ma $\Delta p = \Delta q = \frac{1}{\sqrt{2}}$

quadrature ⊥



lo stato squeezed ha quadrature diverse

st è squeezed $\overset{def}{\iff}$ $\exists A$ t.c. $\Delta A^2 < \frac{1}{2} |\langle [A,B] \rangle|$

Heis - Robertson

$$\Delta A^2 \Delta B^2 \geq \frac{1}{4} |\langle [A,B] \rangle|^2$$

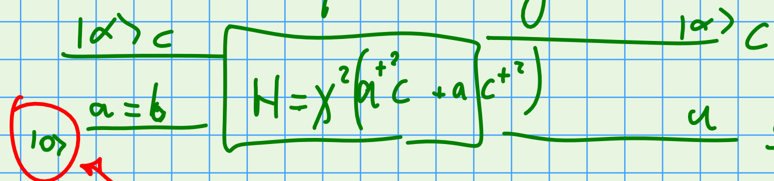
st squeezed ideale $\overset{def}{\iff}$ st a minima indet

$$\Delta A^2 \Delta B^2 = \frac{1}{4} |\langle [A,B] \rangle|^2$$

Op di SQUEEZING $\boxed{S(\xi) \overset{def}{=} e^{\frac{1}{2}(\xi^* a^2 - \xi a^{+2})}}$

è op unitario $S^+ = S^{-1}$: è un op evoluzione unitaria $e^{-\frac{i}{\hbar}Ht}$ con $H \propto i(\xi^* a^2 - \xi a^{+2})$

Hamilt di Squeezing → cristallo non lineare $\chi^{(2)}$



$S(\xi)|0\rangle = |\text{vuoto squeezed}\rangle$

BCH di $su(1,1)$: $K_+ \overset{def}{=} \frac{a^{+2}}{2} = K_-^+$

$$K_z \overset{def}{=} \frac{1}{2}\left(a^+a + \frac{1}{2}\right)$$

soddisfa algebra di $su(1,1)$

$$[K_+, K_-] = \frac{1}{4}[a^{+2}, a^2] = \frac{1}{4}\left(a^+[a^+, a^2] + [a^+, a^2]a^+\right) =$$

$$= \frac{1}{4}\left(a^+a[a^+, a] + a^+[a^+, a]a + [a^+, a]aa^+ + a[a^+, a]a^+\right)$$

$$= -\frac{1}{4}\left(a^+a + a^+a + \underset{a^+a+1}{aa^+} + \underset{a^+a+1}{aa^+}\right) = -a^+a - \frac{1}{2} = -2K_z$$

$$[K_z, K_\pm] = K_\pm$$

$\Rightarrow$ BCH di $SU(1,1)$

$$S(\xi) = e^{\frac{\xi^*}{2}a^2 - \frac{\xi}{2}a^{+2}} \overset{\substack{BCH\\ SU(1,1)}}{=} e^{-\frac{z}{2}a^{+2}} \left(ch|\xi|\right)^{-a^+a-\frac{1}{2}} e^{\frac{\xi^* a^2}{2}}$$

$$z = \frac{\xi}{|\xi|} th|\xi|$$

$\xi \overset{def}{=}$ PARAMETRO di SQUEEZING $\in \mathbb{C}$

VUOTO SQUEEZED $\quad S(\xi)|0\rangle = e^{-\frac{z a^{+2}}{2}}\left(ch|\xi|\right)^{-a^+a-\frac{1}{2}} \cancel{e^{\frac{\xi^* a^2}{2}}}|0\rangle$

$$= \frac{1}{\sqrt{ch|\xi|}} \sum_{n=0}^{\infty} \frac{(-z)^n}{n!} \underbrace{a^{+2n}|0\rangle}_{\sqrt{(2n)!}\,|2n\rangle} = \frac{1}{\sqrt{ch|\xi|}} \sum_{n=0}^{\infty} \sqrt{\binom{2n}{n}} (-z)^n |2n\rangle$$

$$\hookrightarrow \frac{\sqrt{(2n)!}}{n!} = \sqrt{\frac{(2n)!}{n!\,n!}}$$

solo i termini pari nella base di Fock

il vuoto squeezed ha energia:

$$\langle a^+a\rangle = \left(\langle 0|S^+\right)a^+a\left(S|0\rangle\right) = sh^2|\xi| = 0 \Leftrightarrow \xi = 0$$

squeezed coherent st $\quad S(\xi)|\alpha\rangle = S(\xi)D(\alpha)|0\rangle$

displaced squeezed vacuum $\quad D(\alpha)S(\xi)|0\rangle$

VARIANZA delle QUADRATURE del VUOTO SQUEEZED

$\rightarrow$ vedremo che vuoto squeezed è st squeezed

" " è st a minima indet

$$\Delta q\,\Delta p = \frac{1}{2} \qquad \Delta q = \frac{e^{-|\xi|}}{\sqrt{2}} < \frac{1}{\sqrt{2}} = \sqrt{\frac{1}{2}\left|\langle \underbrace{[A,B]}_{i}\rangle\right|} = \frac{1}{\sqrt{2}}$$

$$\Delta p = \frac{e^{+|\xi|}}{\sqrt{2}}$$

$$\Delta a_\varphi^2 = \langle a_\varphi^2 \rangle - \langle a_\varphi \rangle^2 \qquad a_\varphi = \frac{a^+ e^{i\varphi} + a e^{-i\varphi}}{2}$$

$$q = a_{\varphi=0} \qquad p = a_{\varphi=\frac{\pi}{2}}$$

$$\langle X \rangle = \overbrace{\left(\langle 0| S^+\right) X \left(S|0\rangle\right)}^{\text{Schrödinger}} = \overbrace{\langle 0| \left(S^+ X S\right) |0\rangle}^{\text{Heis}}$$

$$S^+ a_\varphi S = \underline{S^+ a^+ S\, e^{i\varphi} + S^+ a S\, e^{-i\varphi}}$$

$$e^A B e^{-A} = B + [A,B] + \dots \frac{1}{n!}[A,[A,\dots[A,B]]] + \dots$$

$$S^+ = e^{-\frac{1}{2}(\xi^* a^2 - \xi a^{+2})}$$

$$S^+ a S = a + \underbrace{\left[-\frac{\xi^* a^2}{2} + \frac{\xi a^{+2}}{2}, a\right]}_{= -\xi a^+} + \frac{1}{2!}\left[-\frac{\xi^* a^2}{2} + \frac{\xi a^{+2}}{2}, -\xi a^+\right]$$

$$\xi^* \xi a \qquad [B,[B,A]]$$

$$+ \frac{1}{3!}\underbrace{\left[-\frac{\xi^* a^2}{2} - \frac{\xi a^{+2}}{2}, |\xi|^2 a\right]}_{-\xi |\xi|^2 a^+} + \dots = \sum_n \frac{|\xi|^{2n}}{(2n)!} a - a^+ \frac{|\xi|^{2n+1}}{(2n+1)!} \frac{\xi}{|\xi|}$$

$$= a\,\mathrm{ch}|\xi| - a^+ \xi \frac{\mathrm{sh}|\xi|}{|\xi|} = S^+ a S$$

$$S^+ a^+ S = (S^+ a S)^+ = a^+ \mathrm{ch}|\xi| - \frac{\xi^*}{|\xi|} a\, \mathrm{sh}|\xi|$$

$$\langle a_\varphi \rangle = \frac{\langle a \rangle e^{-i\varphi} + \langle a^+ \rangle e^{i\varphi}}{2} = \frac{\langle 0|S^+ a S|0\rangle e^{-i\varphi} + e^{i\varphi}\langle 0|S^+ a^+ S|0\rangle}{2}$$

$$= \langle 0| \left(a\, ch|\xi| - a^{+} \frac{\xi}{|\xi|} sh|\xi| \right) |0\rangle \frac{e^{-i\varphi}}{2} + \underbrace{\ldots}_{=0}$$

$$\langle a_\varphi \rangle = 0 \Rightarrow \langle a_\varphi \rangle^2 = 0$$

$$\Delta a_\varphi^2 = \langle a_\varphi^2 \rangle = \frac{1}{2} \langle (a^+ e^{i\varphi} + a e^{-i\varphi})^2 \rangle =$$

$$\frac{1}{2}\left(\langle a^{+2} \rangle e^{2i\varphi} + \langle a^2 \rangle e^{-2i\varphi} + \langle a^+ a \rangle + \langle a a^+ \rangle \right)$$

$\langle a a^+ \rangle = \langle a^+ a \rangle + 1$

$$\langle a^2 \rangle = \langle 0| S^+ a^2 S |0\rangle = \langle 0| S^+ a \underbrace{S S^+}_{=1} a S |0\rangle =$$

$$= \langle 0| \left(a\, ch|\xi| - a^+ \frac{\xi}{|\xi|} sh|\xi| \right)^2 |0\rangle =$$

$$= \langle 0| a^2 \ldots + a^{+2} \ldots - \underbrace{a a^+}_{a^+ a + 1} \frac{\xi}{|\xi|} ch|\xi| sh|\xi| - a^+ a \ldots |0\rangle$$

$= 0$, $= 0$, $= 0$

$$= -\frac{\xi}{|\xi|} ch|\xi| sh|\xi| = \langle a^2 \rangle$$

$$\langle a^{+2} \rangle = \langle a^2 \rangle^* = -\frac{\xi^*}{|\xi|} ch|\xi| sh|\xi|$$

$$\langle a^+ a \rangle = \langle 0| S^+ a^+ a S |0\rangle = \langle 0| S^+ a^+ \underbrace{S S^+}_{=1} a S |0\rangle =$$

numero medio di fotoni del vuoto squeezed

$$= \langle 0| \left(a^+ ch - \frac{\xi^*}{|\xi|} a\, sh \right) \left(a\, ch - \frac{\xi}{|\xi|} a^+ sh \right) |0\rangle$$

$$= \langle 0| \cancel{a^{+2}} \ldots + \cancel{a^2} \ldots + \cancel{a^+ a} \ldots + \underbrace{a a^+}_{a^+ a + 1} \frac{|\xi|^2}{|\xi|^2} sh^2|\xi| \, |0\rangle$$

$= sh^2|\xi| = \langle a^+ a \rangle$

$$\Delta a_\varphi^2 = \langle a_\varphi^2 \rangle = \frac{1}{2}\Big(-ch|\xi| sh|\xi| \underbrace{\frac{\xi^* e^{2i\varphi} + \xi e^{-2i\varphi}}{|\xi|}}_{=2} + 2sh^2|\xi| + 1 \Big)$$

scelgo φ oppure ξ in modo t.c.

$$\frac{\xi^*}{|\xi|} e^{2i\varphi} = +1$$

$$\Delta a_\varphi^2 = \frac{1}{2}\left(+ 2\, ch|\xi|\, sh|\xi| + 2\, sh^2|\xi| + 1 \right) =$$

$ch\,x = \frac{e^x + e^{-x}}{2}$

$sh\,x = \frac{e^x - e^{-x}}{2}$

$$= \frac{1}{2}\left(+ 2 \frac{e^{+x} + e^{-x}}{2} \frac{e^x - e^{-x}}{2} + 2\left(\frac{e^x - e^{-x}}{2}\right)^2 + 1 \right) =$$

$$= \frac{1}{2} e^{2|\xi|} = \Delta a_\varphi^2 = \Delta p^2$$

stesso conto con $\varphi + \frac{\pi}{2}$ $\quad \frac{\xi^* e^{2i\varphi}}{|\xi|} = +1 \Rightarrow \frac{\xi^* e^{2i(\varphi + \frac{\pi}{2})}}{|\xi|} = -1$

$$\Delta a^2_{\varphi + \frac{\pi}{2}} = \Delta q^2 = \frac{e^{-2|\xi|}}{2}$$

$$\Delta a_\varphi = \Delta p = \frac{e^{|\xi|}}{\sqrt{2}} \qquad \Delta q = \frac{e^{-|\xi|}}{\sqrt{2}}$$

quadratura è il campo elettrico $q = a_{\varphi=0}$ campo el

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

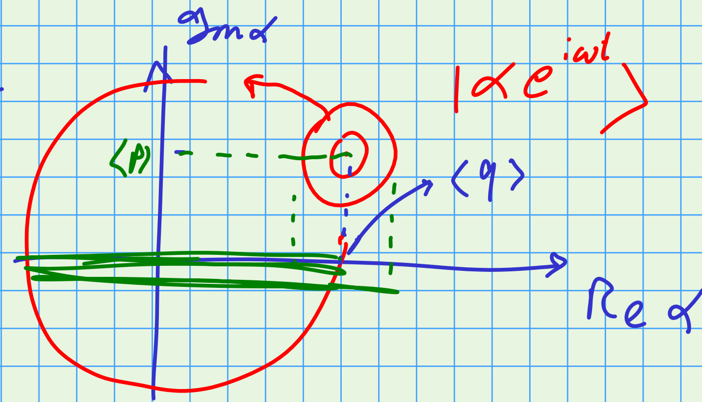
evoluzione temporale della quadratura per evoluz libera $\rightarrow H = \hbar\omega(a^+ a + \frac{1}{2})$ $\qquad a_\varphi^H(t) = e^{\frac{i}{\hbar}Ht} a_\varphi e^{-\frac{i}{\hbar}Ht}$

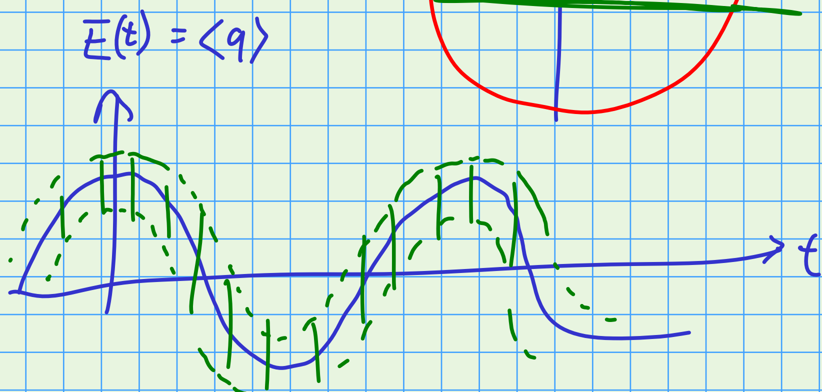
$$= e^{i\omega t a^\dagger a} a_\varphi e^{-i\omega t a^\dagger a} = \frac{1}{\sqrt{2}}\Big(\underbrace{e^{i\omega t a^\dagger a} a e^{-i\omega t a^\dagger a}}_{a e^{-i\omega t}} e^{-i\varphi} + h.c.\Big)$$

$$= \frac{1}{\sqrt{2}}\left(a^\dagger e^{i(\varphi+\omega t)} + a e^{-i(\varphi+\omega t)}\right)$$

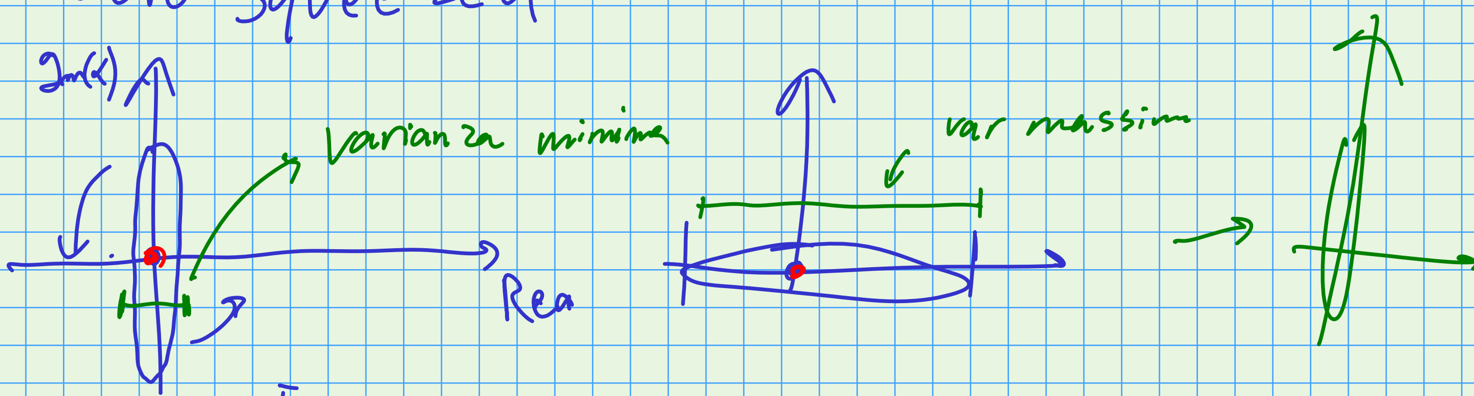
la quadratura ruota con periodo ω

Stato coerente





Vuoto squeezed



Coerente squeezed