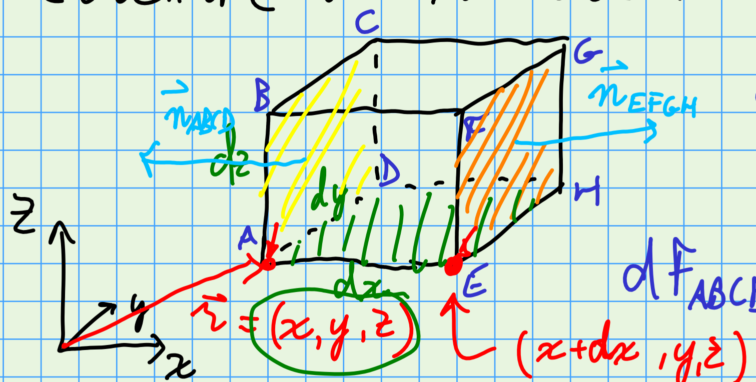
FISICA 2 8/10/20

$$F_{TOT} = \int_{S=\partial V} d^2r \; \vec{n}\cdot\vec{v} = \int_V d^3r \; \vec{\nabla}\cdot\vec{v} \qquad \text{T. di Gauss}$$

cubettare V in cubetti di lato $dx\,dy\,dz$



$$dF_{ABCD} = \underset{\substack{\shortparallel \\ dy\,dz}}{da_{ABCD}} \; \underbrace{\vec{n}_{ABCD}\cdot\vec{v}(x,y,z)}_{-v_x(x,y,z)}$$

$$\vec{n} = -\vec{i}$$

$$dF_{ABCD} = -dy\,dz\,\cancel{v_x(x,y,z)}$$

$$dF_{EFGH} = \underset{\substack{\shortparallel \\ dy\,dz}}{da_{EFGH}} \; \vec{n}_{EFGH}\cdot\vec{v}(x+dx,y,z) =$$

$$= dy\,dz\,v_x(x+dx,y,z) \overset{\text{SST al 1° ordine}}{\simeq} dy\,dz\left[\cancel{v_x(x,y,z)} + dx\frac{\partial v_x(x,y,z)}{\partial x} + dx^2 \ldots\right]$$

$$dF_{\substack{ABCD\\EFGH}} = dF_{ABCD} + dF_{EFGH} \simeq dx\,dy\,dz\,\frac{\partial v_x}{\partial x}(x,y,z)$$

$$dF_{ADEH} + dF_{BCGF} = dx\,dy\,dz\,\frac{\partial v_y}{\partial y}(x,y,z)$$

$$dF_{ABFE} + dF_{CDHG} = dx\,dy\,dz\,\frac{\partial v_z}{\partial z}$$

$$dF_{CUBETTO} = dx\,dy\,dz\,\underbrace{\vec{\nabla}\cdot\vec{v}}_{\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}} = d^3r\,\vec{\nabla}\cdot\vec{v}$$

$$\int_V d^3r\,\vec{\nabla}\cdot\vec{v} = \int dF_{CUBETTO}$$

SOMMO SU TUTTI I CUBETTI

i cubetti interni non contribuiscono all'integr sul flusso

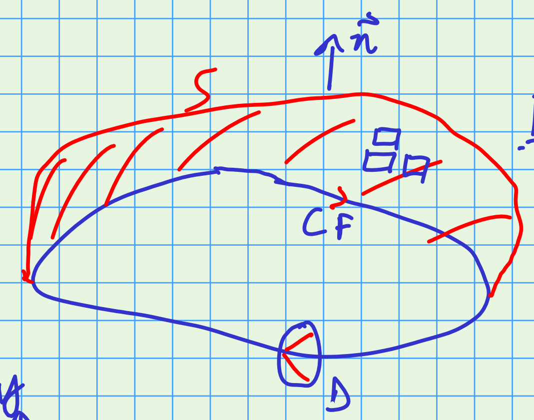
rimangono solo i contributi sulla superficie esterna $S=\partial V$

$$= \int_{S=\partial V} d^2 r \; \vec{n}\cdot\vec{v}$$

LEGGE STOKES

$$\int_{C=\partial S} d\vec{r}\cdot\vec{v}(\vec{r}) = \int_S d^2 r \; \vec{n}\cdot\vec{\nabla}\times\vec{v}(\vec{r})$$

↑ circuito chiuso



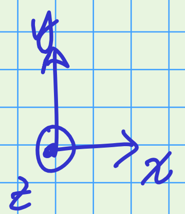
verso di $\vec{n}$ va scelto con la regola della mano dx.

quadrettiamo la sup in quadratini

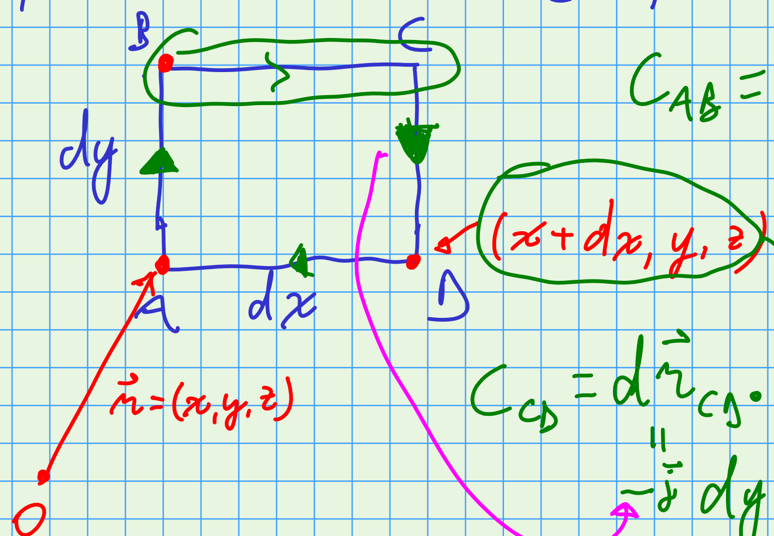
quadratini // al piano xy

" " yz

" " xz



quadratino ⊥ asse z, // a piano xy



$$C_{AB} = \underbrace{d\vec{r}_{AB}}\cdot\vec{v}(x,y,z) = dy\, v_y(x,y,z)$$

vettore che rappres il lato AB

$$d\vec{r}_{AB} = \vec{j}\, dy$$

↑ versore y

$$C_{CD} = d\vec{r}_{CD}\cdot\vec{v}(x+dx,y,z) = -dy\, v_y(x+dx,y,z)$$

$$= -\vec{j}\, dy$$

$$\overset{SST}{\simeq} -dy\left[\cancel{v_y(x,y,z)} + dx\frac{\partial v_y}{\partial x}(x,y,z) + dx^2 \ldots\right]$$

$$C_{BC} = dx\left[\cancel{v_x(x,y,z)} + dy\frac{\partial v_x}{\partial y}(x,y,z)\right]$$

$$C_{DA} = -dx\left[\cancel{v_x(x,y,z)} + dy\cancel{\frac{\partial v_x}{\partial y}(x,y,z)} + \cancel{dy^2} \ldots\right]$$

$$C_{quadratino} = C_{AB} + C_{BC} + C_{CD} + C_{DA} = -dx\,dy\underbrace{\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)}_{(\vec{\nabla}\times\vec{v})_z}$$

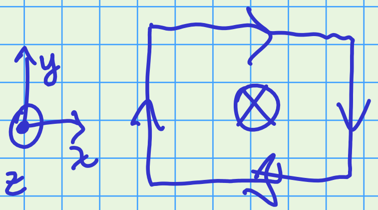
$$= -dx\,dy\;\vec{k}\cdot(\vec{\nabla}\times\vec{v}) =$$

↑ versore // asse z

$$(\vec\nabla\times\vec v)_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}$$

$$= d^2r\ \vec n\cdot(\vec\nabla\times\vec v)$$

$\vec n$ versore $\perp$ al quadratino scelto con la regola mano dx

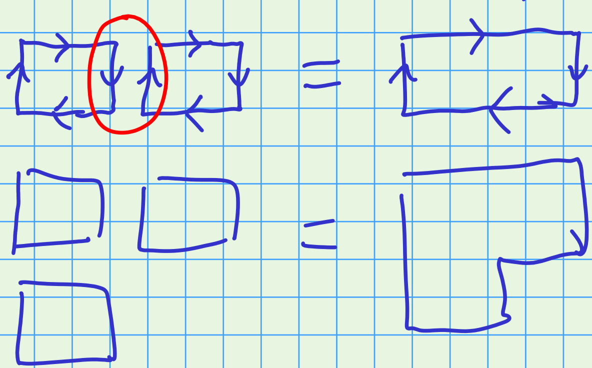


$\vec n = -\vec k$

i quadratini $\perp$ asse x $\rightarrow$ $(\vec\nabla\times\vec v)_x$

$\perp$ " y $\rightarrow$ $(\vec\nabla\times\vec v)_y$

sommiamo su tutti i quadratini che appros S



i lati interni si annullano a coppie e rimane solo il contributo dei quadratini sul bordo

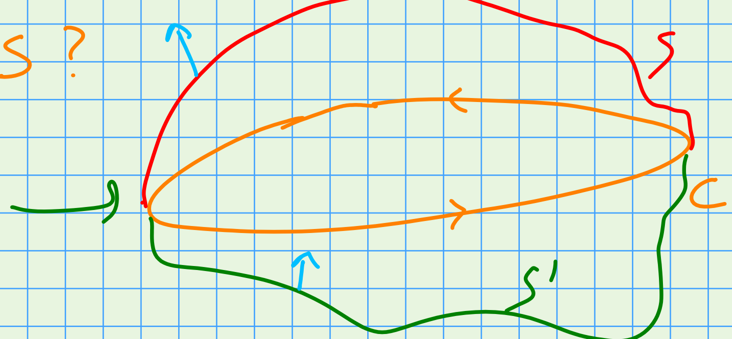
SOMMA della circuitazione di tutti i quadratini

$$= \oint_{C=\partial S} d\vec r\cdot\vec v = \int_S d^2r\ \vec n\cdot\vec\nabla\times\vec v$$

C bordo

estensione del T. Fondamentale del calcolo integ

QUALI S?



è irrilevante

$$\int_S d^2r\ \vec n\cdot\vec\nabla\times\vec v = \int_{S'} d^2r \ldots$$

nel caso di sup chiusa S'' $\int_{S''} d^2r\ \vec n\cdot\vec\nabla\times\vec v = \oint_{\partial S''} d\vec r\cdot\vec v = 0$

S'' chiusa

$$S \cup S' = S'' \qquad \int_{S \cup S'} = \int_{S''} = 0$$

$$\int_S + \int_{S'} \qquad \int_S = -\int_{S'}$$

va via se usiamo regola mano dx

$$\boxed{\int_S d^2r\, \vec{n} \cdot \vec{\nabla} \times \vec{v} = \int_{S'} d^2r\, \vec{n} \cdot \vec{\nabla} \times \vec{v}}$$

GRADIENTE in coord polari sferiche (r, ϑ, φ)

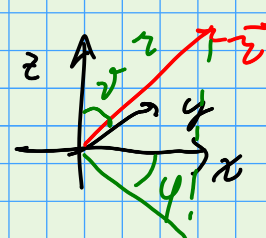
in coord cartes $\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$$\vec{\nabla} = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\vartheta \frac{1}{r} \frac{\partial}{\partial \vartheta} + \vec{e}_\varphi \frac{1}{r \sin\vartheta} \frac{\partial}{\partial \varphi}$$

$\vec{e}_r, \vec{e}_\vartheta, \vec{e}_\varphi$ sono versori identificano le coord polsferi

$\vec{e}_r$ è versore radiale $\vec{e}_r = \frac{\vec{r}}{|\vec{r}|}$ punta verso zenith

r punta nella direz di incremento di $r \to$ modulo

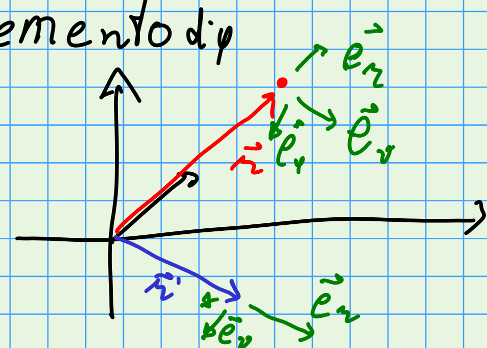


$\vec{e}_\vartheta$ versore punta incremento di ϑ

$\Rightarrow$ punta verso il polo sud

$\vec{e}_\varphi$ versore punto incremento di φ

$\Rightarrow$ punta verso EST



$$\vec{\nabla} f = \vec{e}_r (\vec{\nabla} f)_r + \vec{e}_\vartheta (\vec{\nabla} f)_\vartheta + \vec{e}_\varphi (\vec{\nabla} f)_\varphi$$

$$df = \vec{\nabla} f \cdot d\vec{r} = dx \frac{\partial f}{\partial x} + dy \frac{\partial f}{\partial y} + dz \frac{\partial f}{\partial z}$$

↑ coord cartesiane

↑ incremento del campo $f(\vec{r}) \to f(\vec{r}+d\vec{r})$

→ componente x di $d\vec{r}$ $\quad (d\vec{r})_x = dx$

$$= (\vec{\nabla} f)_r (d\vec{r})_r + (\vec{\nabla} f)_\vartheta (d\vec{r})_\vartheta + (\vec{\nabla} f)_\varphi (d\vec{r})_\varphi$$

↑ coor polari

vettore

$$d\vec{r}(r,\vartheta,\varphi) = \frac{\partial \vec{r}}{\partial r} dr + \frac{\partial \vec{r}}{\partial \vartheta} d\vartheta + \frac{\partial \vec{r}}{\partial \varphi} d\varphi$$

L modulo

$$= \vec{e}_r \left|\frac{\partial \vec{r}}{\partial r}\right| dr + \vec{e}_\vartheta \left|\frac{\partial \vec{r}}{\partial \vartheta}\right| d\vartheta + \vec{e}_\varphi \left|\frac{\partial \vec{r}}{\partial \varphi}\right| d\varphi$$

$$\vec{e}_r = \frac{\partial \vec{r}}{\partial r} \Big/ \left|\frac{\partial \vec{r}}{\partial r}\right| \qquad \vec{e}_\vartheta = \frac{\partial \vec{r}}{\partial \vartheta} \Big/ \left|\frac{\partial \vec{r}}{\partial \vartheta}\right|$$

$$= (d\vec{r})_r \vec{e}_r + (d\vec{r})_\vartheta \vec{e}_\vartheta + (d\vec{r})_\varphi \vec{e}_\varphi$$

$$(d\vec{r})_r = dr \left|\frac{\partial \vec{r}}{\partial r}\right| = dr \left|(\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)\right|$$

$$\vec{r} = (r\sin\vartheta\cos\varphi, r\sin\vartheta\sin\varphi, r\cos\vartheta)$$

$$= dr \sqrt{\sin^2\vartheta\cos^2\varphi + \sin^2\vartheta\sin^2 + \cos^2\vartheta} = dr$$

$$(d\vec{r})_\vartheta = d\vartheta \left|\frac{\partial \vec{r}}{\partial \vartheta}\right| = d\vartheta \left|(r\cos\vartheta\cos\varphi, +r\cos\vartheta\sin\varphi, -r\sin\vartheta)\right|$$

$$= d\vartheta \sqrt{r^2(\cos^2\vartheta\cos^2\varphi + \cos^2\vartheta\sin^2\varphi + \sin^2\vartheta)} = r\, d\vartheta$$

$$(d\vec{r})_\varphi = d\varphi \left|\frac{\partial \vec{r}}{\partial \varphi}\right| = d\varphi \; r|\sin\vartheta|$$